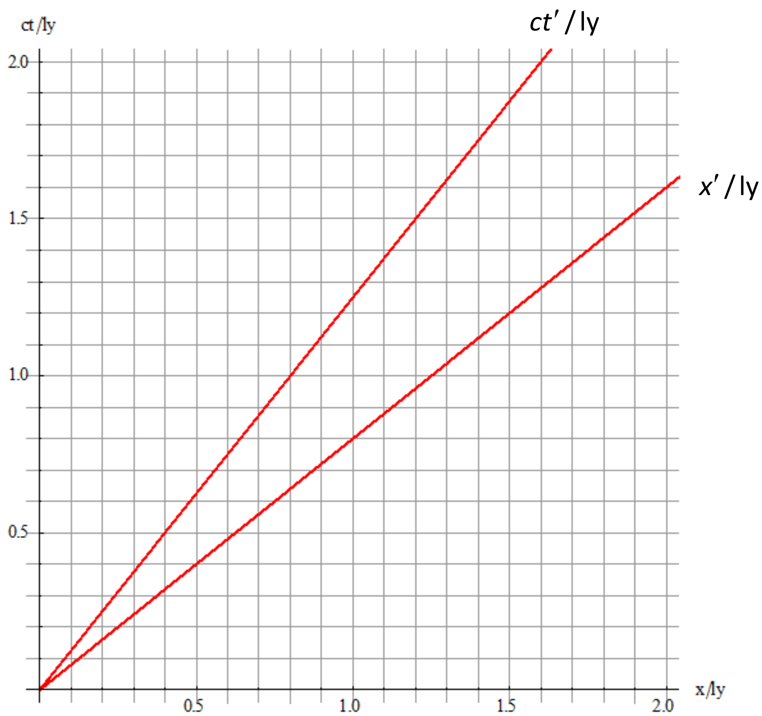
Teacher notes
Topic A

The scale on the two sets of axes is not the same.

Where are the “ones” on the primed axes?

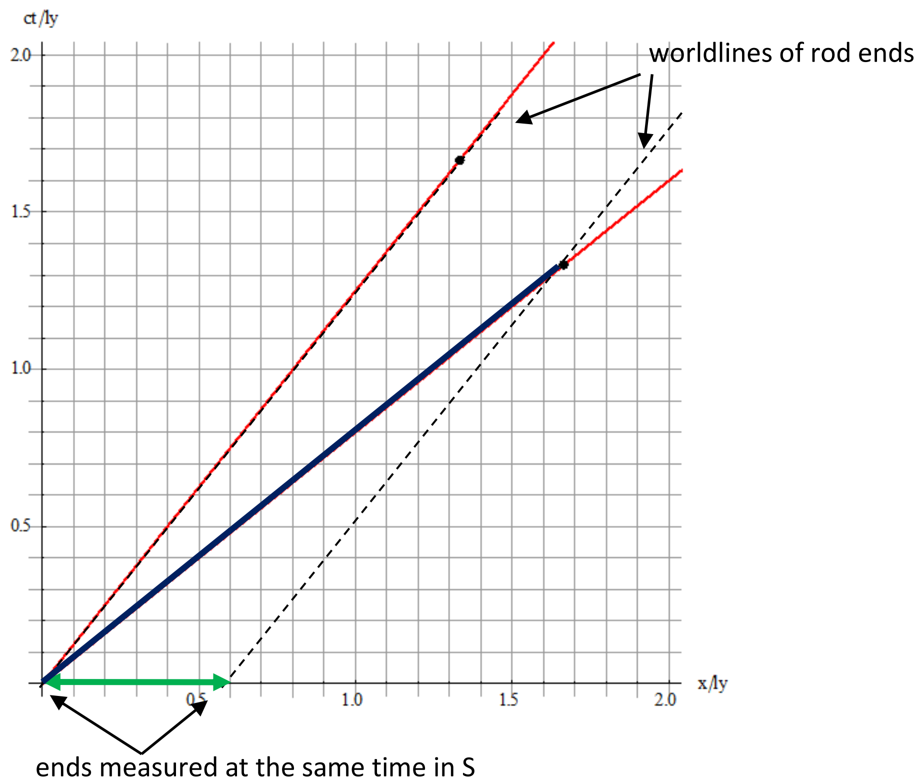


The point we are looking for on the primed x-axis has coordinates: $(x'=1, ct'=0)$. In the S frame this gives $x=\gamma(x'+vt')=\gamma$. Similarly, the point on the primed time axis has coordinates $(x'=0, ct'=1)$. This means that $ct=\gamma(ct'+\frac{v}{c}x')=\gamma$. Thus we need to find the gamma factor.

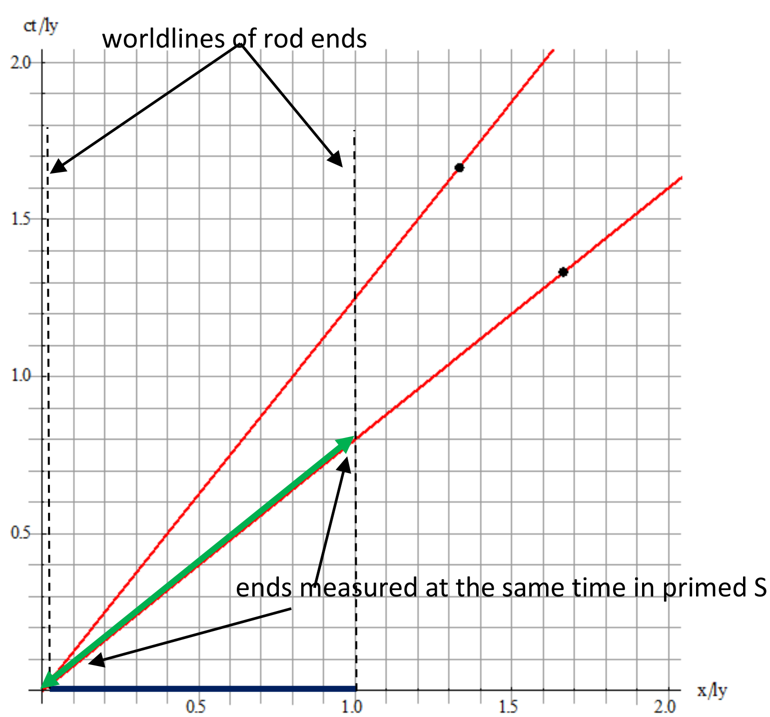
The speed is 0.80 c and so $\gamma=1.667$.

Once the scale is determined we can examine the usual phenomena of length contraction and time dilation easily and also see that these are symmetric phenomena.

The diagram shows a rod of proper length 1 ly that is at rest in frame S'. An observer in S must measure the position of the ends of the rod at the same time (according to S clocks) in order to determine the length of the rod. The S observer measures a length of 0.6 ly, a contracted length from the proper length of 1 ly (green double headed arrow).



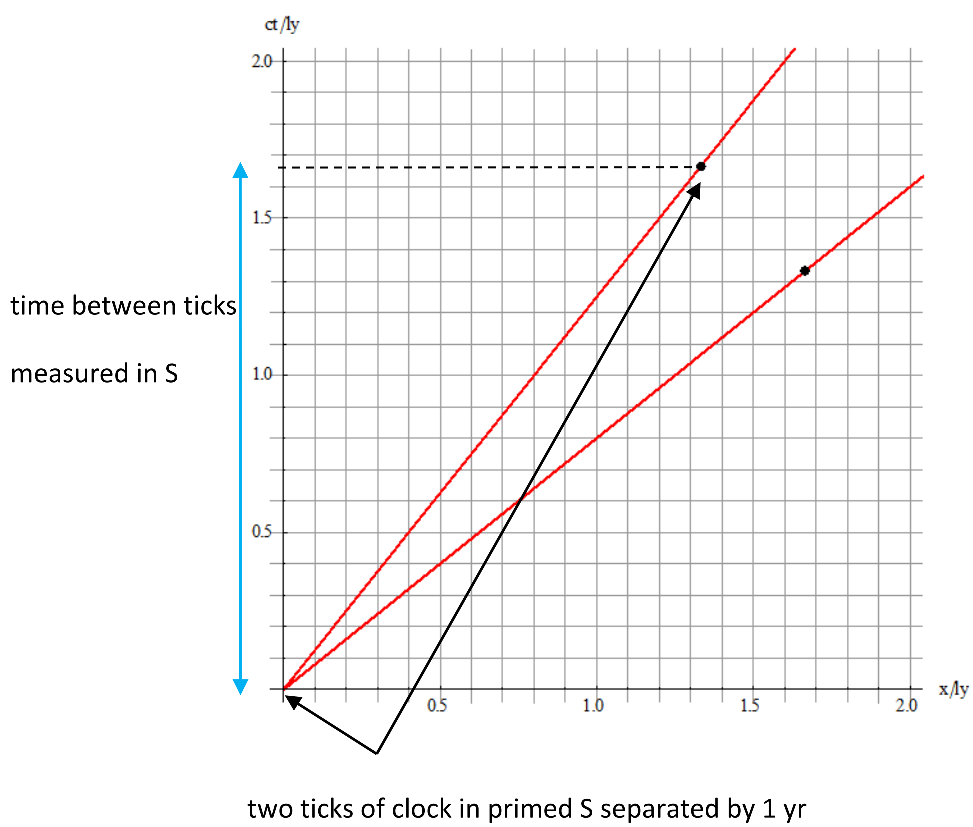
Similarly, a rod of proper length 1 ly is at rest in S.

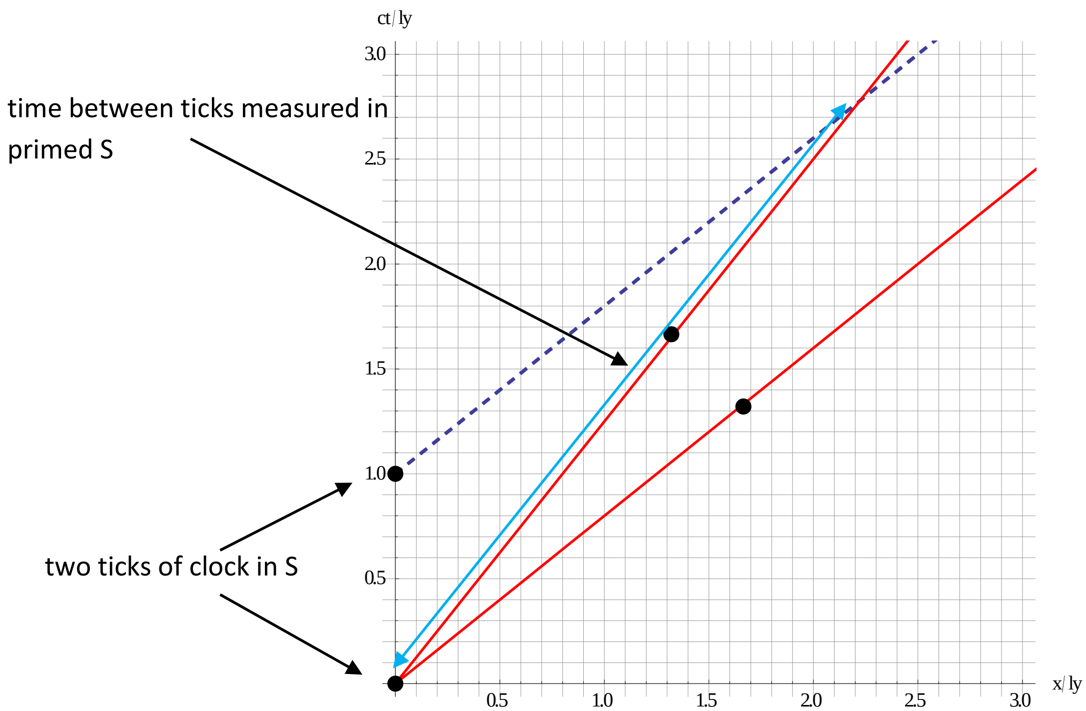


The S' observer measures the end of the rods at the same time (according to S' clocks). The measured length is 0.6 ly.

So length contraction is a symmetric effect.

You can now discuss the symmetric nature of time dilation in the same way.



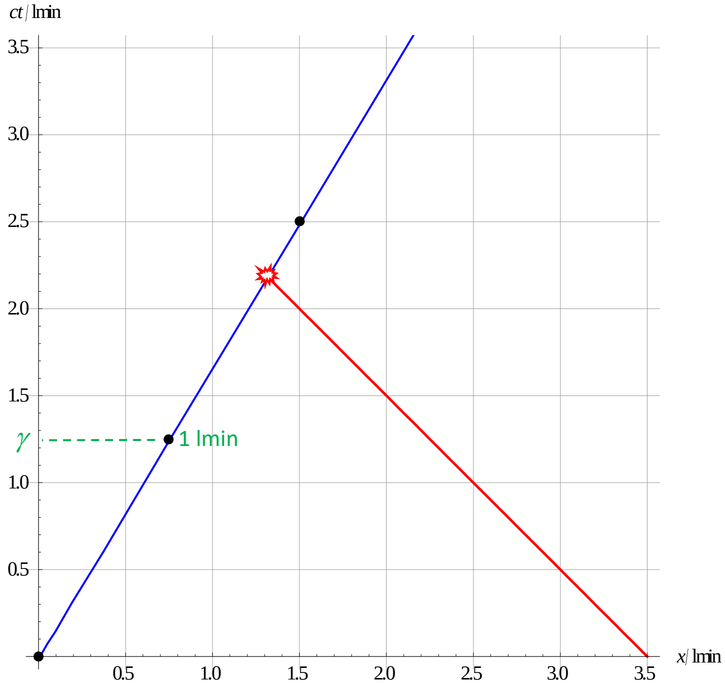


Spacetime diagrams can easily solve problems like this:

An alien cruise ship moves past earth with speed 0.60 c. A laser beam is directed towards the ship from a space station 3.5 lmin (a light minute, lmin, is the distance travelled by light in one minute) from earth. The beam is launched just as the ship moves past earth. The alien ship, realizing that it will come under attack, raises a shield just it passes earth. It takes two minutes by ship clocks to activate the shield.

Will earth shoot down the alien craft?

Answer The diagram shows the worldline of the alien ship.



The red line is the worldline of a photon launched at t = 0 from x =3.5 lmin. The gamma factor is 1.25 which allows us to find the scale on the ship worldline: two dots are separated by 1 minute by ship clocks. The photons intercept the alien ship before two minutes go by according to alien ship clocks so their shield is not operational and the alien ship will be destroyed.

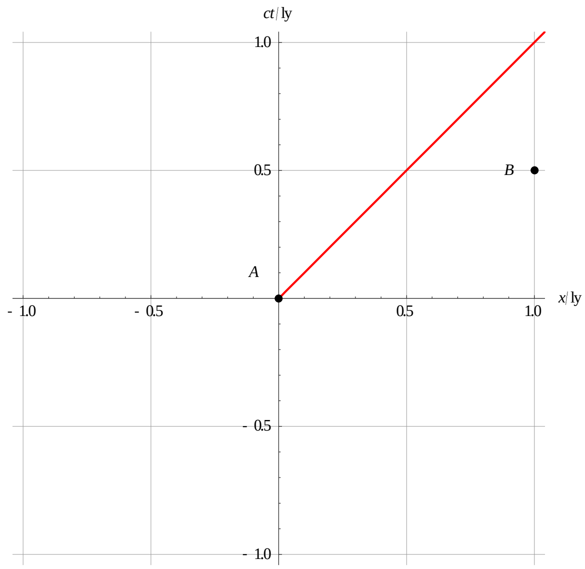
From the diagram it appears that the laser beam arrives at the cruise ship at a time of 1.75 min according to the ship. How do we derive this number? According to earth the distance between the ship and the launch point is 3.5 lmin so it will take a time $\frac{3.5}{1.6} = 2.1875 \text{ min}$ which we also see from the diagram. The reason for the factor of 1.60 is that the distance between ship and launch point is decreasing at a rate of c + 0.6c = 1.6c. There is no problem with the postulate of relativity about the speed of light being the limiting speed. No material object is moving at this speed. For the ship we use a Lorentz transformation: event 1 is the launch and event 2 is the arrival of the beam at the ship. For these events: $x_1 = 3.5,\quad t_1 = 0$ and $x_2 = 2.2875 \times 0.60 = 1.3125,\quad t_2 = 2.1875$. Hence,

$$c\Delta t' = \gamma(c\Delta t - \frac{v}{c}\Delta x) = 1.25 \times (2.1875 - 0.60 \times 1.3125) = 1.75 \text{ min}.$$

The use of spacetime diagrams is so much easier.

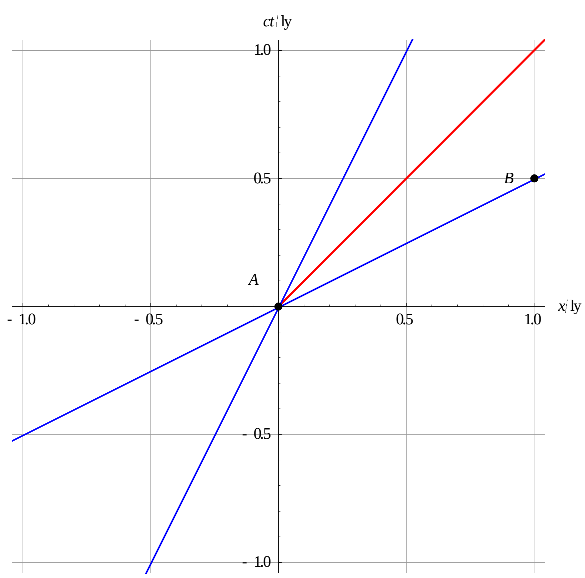
Causality

Consider also this problem: the diagram shows two events, A and B and a photon worldline originating at $x = 0$. Clearly, B occurs after A as far as frame S is concerned. Event B is outside the light cone at A and so event A **cannot be the cause** of event B.



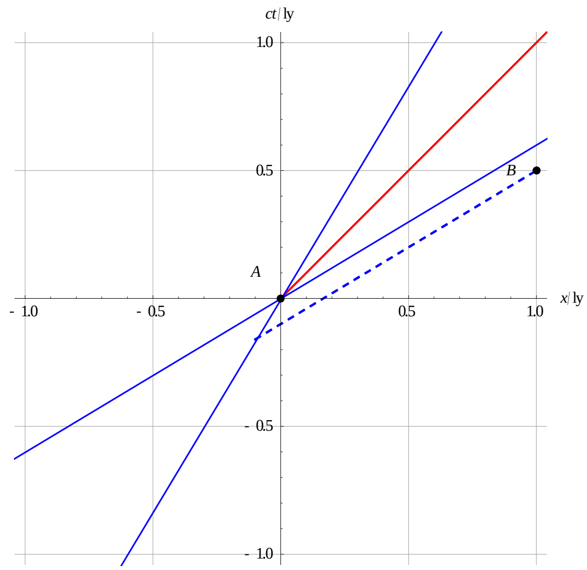
(We would need a speed of 2c to influence B from A.)

Now consider another frame, S', going past S. If the speed of this frame with respect to S is 0.5c we would have this diagram for the spacetime axes of S'.



We see that in S' events A and B are simultaneous!

In a frame moving with any speed greater than 0.5 c, say 0.6 c we would have:

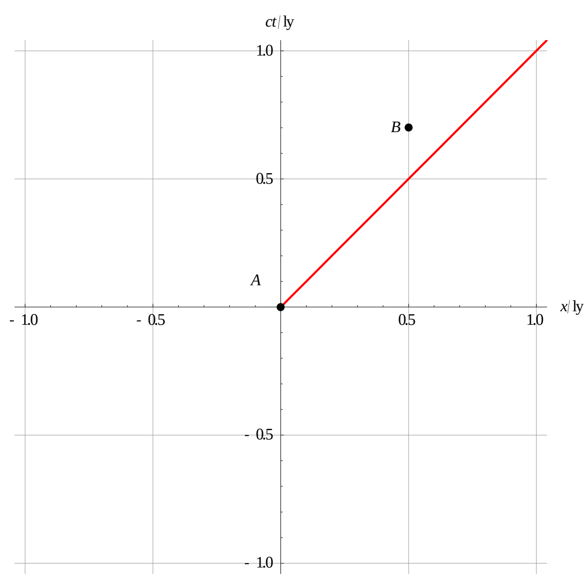


We see that in this frame, **B occurs before A**!

This would have been a fatal problem if A was the cause of B, but as we saw this is not the case.

Let us verify that, indeed, if A is the cause of B, then no frame can be found in which B occurs **before** A.

Consider events A and B now:



For A and B to be simultaneous the space axis of a frame S' would have to be a line joining A and B and would thus make an angle with the *x* axis greater than 45°. The angle θ the S' frame space axis makes with the *x* axis is given by $\tan\theta = \frac{v}{c}$ and if $\theta > 45^\circ$, then $v > c$. This is impossible! And so, it is just as impossible to find a frame in which B occurs before A.